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Library.

The Council desire to thank Colonel de Villamil for a copy of "The Principle of Least Action," by Philip E. B. Jourdain, and Prof. G. H. Bryan for a copy of "On the Equations of Motion of a Plane Surface through Air, with special reference to its stability (Constrained Motion of a Plane Kite)," by Prof. J. M. Bose, M.A.

Edward Busk Memorial Lecture.

The first Edward Busk Memorial Lecture will be read by Mr. F. W. Lanchester, M.Inst.C.E., on a date in the New Year, to be subsequently fixed.

Times of Opening of the Society's Offices.

Until further notice, from 10.0 a.m. to 12 noon on Monday, Tuesday, Wednesday, Thursday, and Friday. Closed on Saturday.

Reviews and Obituary.

Temporarily held over.

THE SMALL OSCILLATIONS OF A KITE.

BY PROF. G. H. BRYAN, F.R.S.

I.—INTRODUCTION.

1. A mathematical investigation of the small oscillations of a kite was proposed in "Stability in Aviation" in the form of a problem (p. 180, problem 16). A solution of this problem has been given by Prof. J. M. Bose in the Bulletin of the Calcutta Mathematical Society, Vol. II., No. 1. Unfortunately his investigation contains many serious errors of a fundamental character, the effect of which is to render the solution inapplicable to a system in any way resembling an ordinary kite. As examples, the variations in the components of the tension of the kite string are assumed to depend on the velocity components instead of on the displacements of the kite, although Prof. Bose's previous equations show the contrary to be the case. Again, while in his introduction he considers the case of a kite attached by a forked string he contradicts himself by neglecting the displacements of the point of intersection of the string with the plane of the kite. Further, in §§13, 14, he omits one of the six equations of motion which he has just written down. The paper contains other errors as well.

Mr. Berwick and I carefully examined Prof. Bose's paper, but came to the conclusion that nothing short of a complete re-investigation of the equations of motion would meet the case.

The main difficulty which distinguishes the problem of the kite from that of the aeroplane is connected with the action of the kite string. If the kite is attached by means of a forked string in the form of a Y, the three independent axes about which it can rotate without displacing the string are not all concurrent, so that even the kinematical conditions would become complicated in the case of a general solution. As it is uncertain how far it would be worth while to give such a solution the present paper is limited to the consideration of the small oscillations of the kite about its position of equilibrium. In this case the expressions for the linear and angular accelerations for fixed and moving axes become the same, and, what is more important, the conditions introduced by the forked string may be satisfied by treating the point of attachment as being different for the longitudinal and lateral oscillations. The simplest plan is then to use a different origin and axes in studying the two types of oscillation.

In this paper the equations are formulated for the general case in which the string is of finite length and extensible, and particular modifications occur when the string is inextensible and when it is practically infinite in length. Any attempt to take into account the weight and therefore also the inertia of the string would necessarily introduce the small oscillations of the string itself about its form of equilibrium in a catenary, and perhaps the wind resistance on the string would also require consideration. It might however be possible to make some more or less satisfactory assumption that would represent these effects, for example, inertia of the string might be represented by components of tension proportional to the accelerations of the point of attachment.

At the time when "Stability in Aviation" was written it seemed probable that it might be necessary to determine the stability coefficients of aeroplanes by attaching them to strings, flying them as kites and observing their periods of oscillation. The main difficulty is, of course, that the disturbances produced by

fluctuations in wind velocity are calculated to render the free oscillations difficult of observation. Some preliminary mathematical work was done on the lines of the present paper by Prof. (now Lieut.) E. H. Harper, M.A., when my book was in preparation, and he obtained biquadratic equations for the oscillations. According to the present investigation the resulting equations may be of the sixth degree for longitudinal and of the fifth degree for lateral oscillations. The reason for the difference in this respect as compared with the corresponding equations for the aeroplane is, generally speaking, that, owing to the action of the string and the wind, the kite can only occupy a definite position of equilibrium, whereas the equilibrium of the aeroplane is not to the same extent dependent on its position in space.

It is convenient to start with the general case of a dynamical system possessing no special kind of symmetry, because in this case the equations of motion become symmetrical. As soon as considerations of symmetry are applied to a dynamical system, the symmetry usually disappears from its equations of motion.

I have purposely introduced the method of cross multiplication in writing down some of the expressions, because I find it of very great use in writing down the equations of motion of a rigid body or the six components of a system of forces, and it is well that anyone who is obliged to make use of these formulæ should know of this simple and convenient method of writing them down.

It has been assumed that the wind is blowing in a horizontal direction. If this be not the case, the necessary modifications in the equations should present no difficulty.

II.—GENERAL EQUATIONS OF SMALL OSCILLATION. ORIGIN CENTRE OF MASS.

2. Take axes fixed in the body (the kite) through its centre of mass. Suppose the string is attached at a single point, and let the notation be defined as follows:—

Tension:—x, y, z, co-ordinates of point of attachment.
S_x, S_y, S_z, components of tension referred to directions which remain fixed *in space*, s length of string.

Gravity:—l, m, n, direction cosines of gravity, *i.e.*, of the vertical.

Air Pressure:—V_1, V_2, V_3, components of the velocity with which the wind is blowing *against* the kite, or algebraically, $-U_1$, $-U_2$, $-U_3$, the components in the positive direction of the axes.
$-X$, $-Y$, $-Z$, $-L$, $-M$, $-N$, forces and couples due to air pressure as in "*Stability in Aviation*," so that X represents a force in the negative direction.

Displacements:—Let the body receive small displacements.
ξ, η, ζ, displacements of the centre of gravity.
u, v, w, corresponding velocities.
θ_1, θ_2, θ_3, angular displacements (small) about the axes.
p, q, r, corresponding angular velocities.

Inertia Coefficients:—W, weight of kite (say in lbs. weight).
A, B, C, moments, D, E, F, products of inertia about axes in lbs. $\times$ ft.^{2}

3. **Changes in constants due to displacement.**—In consequence of the rotation of the axes the above vectors representing the components of tension, gravity and wind velocity will be altered in the displaced position, and the changes can be written down by the method of cross multiplication in the usual way. If α, β, γ,

be components of any such vector, the cross multiplication for transforming from the old to the new axes stands thus:—

Old

$$\begin{matrix} \alpha & & \beta & & \gamma & & \alpha \\ \theta_1 & \times & \theta_2 & \times & \theta_3 & \times & \theta_1 \end{matrix}$$

New

$$\alpha^1 = \alpha + \beta\theta_3 - \gamma\theta_2 \qquad \beta^1 = \beta + \gamma\theta_1 - \alpha\theta_3 \qquad \gamma^1 = \gamma + \alpha\theta_2 - \beta\theta_1$$

Thus, for the new components of

Tension $Sx + \theta_3 Sy - \theta_2 Sz$ . . . . . (1)
Gravity $W(l + \theta_3 m - \theta_2 n)$ in lbs. weight . . . (2)
Wind Velocity $V_1 + \theta_3 V_2 - \theta_2 V_3$ . . . . . (3)

The variations of the air resistance will thus contain terms in the resistance derivatives due to the angular displacement as well as to the velocity components, and they assume the form

$$X = X_0 + (u + \theta_3 V_2 - \theta_2 V_3) X_u + \text{etc.} \ldots + rX_r \quad . \quad . \quad (4)$$
$$N = N_0 + (u + \theta_3 V_2 - \theta_2 V_3) N_u + \ldots + rN_r \quad . \quad . \quad (5)$$

For the displacement in space of the point of attachment, the rule of cross multiplication reads the reverse way.

$$\begin{matrix} \theta_1 & & \theta_2 & & \theta_3 & & \theta_1 \\ x & \times & y & \times & z & \times & x \end{matrix}$$

$$\xi^1 = \xi - \theta_3 y + \theta_2 z \qquad \eta^1 = \eta - \theta_1 z + \theta_3 x \qquad \zeta^1 = \zeta - \theta_2 x + \theta_1 y$$

For the moments of the tension about the centre of gravity the rule of cross multiplication reads

$$\begin{matrix} \text{Point of application} & & x & & y & & z \\ \text{Force} & & S_x + \theta_3 S_y - \theta_2 S_z & \times & S_y + \theta_1 S_z - \theta_3 S_x & \times & S_z + \theta_2 S_x - \theta_1 S_y \end{matrix}$$

Couples

$$\left\{ \begin{matrix} y(S_z + \theta_2 S_x - \theta_1 S_y) \\ -z(S_y + \theta_1 S_z - \theta_3 S_x) \end{matrix} \right\} \left\{ \begin{matrix} z(S_x + \theta_3 S_y - \theta_2 S_z) \\ -x(S_z + \theta_2 S_x - \theta_1 S_y) \end{matrix} \right\} \left\{ \begin{matrix} x(S_y + \theta_1 S_z - \theta_3 S_x) \\ -y(S_x + \theta_3 S_y - \theta_2 S_z) \end{matrix} \right\} \quad (6)$$

4. *The linear and angular components of the mass accelerations* when expressed in lbs. weight are *in the case of small oscillations only*.

$$W\frac{du}{gdt} \text{ or } W\frac{d^2\xi}{gdt^2} \quad , \quad W\frac{dv}{gdt} \text{ or } W\frac{d^2\eta}{gdt^2} \quad , \quad W\frac{dw}{gdt} \text{ or } W\frac{d^2\zeta}{gdt^2} \qquad (7)$$

and

$$A\frac{dp}{gdt} - F\frac{dq}{gdt} - E\frac{dr}{gdt} \text{ or } A\frac{d^2\theta_1}{gdt^2} - F\frac{d^2\theta_2}{gdt^2} - E\frac{d^2\theta_3}{gdt^2} \quad . \qquad (8)$$

and similar expressions for the corresponding couples about the other axes.

To write down the equations of motion it is only necessary to equate the following:—

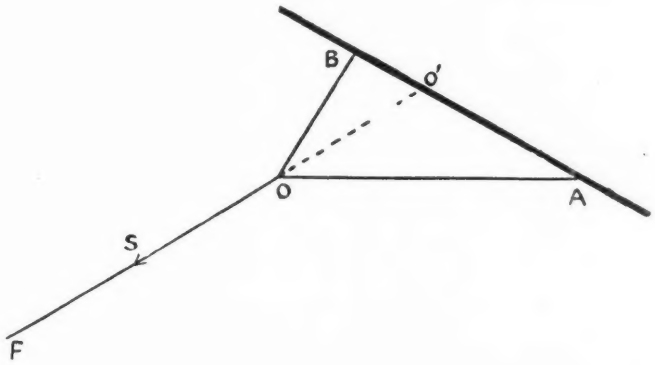
Linear mass acceleration of (7) = { Tension component of (1). + gravity component of (2). — air resistance component of (4). (9)

Angular mass acceleration of (8) = { Tension moment of (6). — air resistance moment of (5). (10)

and there would be no advantage in our spelling these equations out in full. It will be observed that these equations necessarily contain the angular displacements θ_1, θ_2, θ_3, as well as the corresponding velocities p, q, r, and therefore

they may be expected in the general case to lead to period equations of higher degree than is the case with the aeroplane.

III.—ACTION OF THE STRING.



5. The character of the oscillations will depend very largely on the mode of attachment of the string and on the manner in which its tension varies when the kite is displaced.

CASE I.—**String weightless and inextensible.**—The simplest cases are those in which the string is assumed to be weightless and inextensible and attached to a fixed point F in the ground. In such cases the kite is perfectly free to rotate about F, but cannot move in the direction of the string.

In many kites, moreover, the attachment is forked as shown in the figure. In these the following displacements are possible:—

1. The kite and string may turn about F, its direction of motion being perpendicular to OF and either in or perpendicular to the plane of the figure. This gives two possible displacements.

2. It may turn about an axis through O perpendicular to the plane of the figure, the string remaining at rest.

3. It may similarly turn about OF.

4. It may turn about the line AB.

Let FO produced meet AB in O'. Then it will be seen that when the string OF remains at rest

O is the centre of rotation for longitudinal displacements,
O' is the centre for lateral displacements.

[The kite is, of course, *able* to turn about any axis through O whatever, but unless the three strings OA, OB, OF, remain in one plane the tensions in them cannot be in equilibrium, and such a rotation is statically impossible unless O is weighted, which is not supposed the case.]

It follows that *the point of attachment of the string must be taken to be O for longitudinal and O' for lateral oscillations.*

If S is the tension of the string and if the point of attachment undergoes a *small* displacement d_1 perpendicular to the string, the length of the string up to the point of attachment being s, it is easily seen that the component of the tension

tending to bring the kite back again is Sd_1/s to the necessary order of approximation. If s is extremely long this component becomes negligible and the equilibrium becomes neutral for displacements perpendicular to the string.

6. CASE II.—**String extensible.**—In this case if the kite is displaced towards F through a distance d_2 the tension is changed from S_0 to $S_0 - Ed_2/s$ where E is the modulus of elasticity.

Now it will be seen that in the case of the inextensible string we have the geometrical condition $d_2 = O$, but that the tension S is indeterminate in the displaced position. Thus, it would be necessary to eliminate S from the equations of motion by means of this geometrical condition. It is clear that with the centre of gravity as origin the equation of moments also becomes very complicated, and therefore the work is greatly simplified by taking as origin the point of attachment, O for longitudinal and O' for lateral oscillations, so that the moment of the tension components about the origin vanishes.

IV.—ORIGIN AT POINT OF ATTACHMENT.

7. In this case the expressions of § II. will be modified as follows:—

$-x$, $-y$, $-z$, will now be co-ordinates of the centre of gravity.

ξ^1, η^1, ζ^1, displacements of point of contact.

V_1, V_2, V_3, will be unaltered, but *it will be necessary to refer the resistance components and resistance derivatives to the new origin. The values of these will therefore be entirely different from those of* § II. and will only be deducible from them by the formulæ of transformation (*Stability in Aviation*, § 30, and corresponding generalisations for space in three dimensions). With this assumption the forms in which the resistance derivatives occur will be the same as in § II.

The couples due to the tension will now vanish, but instead we shall have couples due to gravity which may be written down by the cross multiplication method.

Point of application	$-x$	$-y$	$-z$
Force	$W(l+\theta_3 m-\theta_2 n)$	$W(m+\theta_1 n-\theta_3 l)$	$W(n+\theta_2 l-\theta_1 m)$

Couples $W\{z(m+\theta_1 n-\theta_2 l)-y(n+\theta_2 l-\theta_1 m)\}$ and two similar. (11)

8. Finally the components of mass acceleration of the centre of gravity are *for small oscillations only.*

$$W\left(\frac{d^2\xi^1}{gdt^2}+\frac{y}{g}\frac{d^2\theta_3}{dt^2}-\frac{z}{g}\frac{d^2\theta_2}{dt^2}\right) \text{ or } W\left(\frac{du^1}{gdt}+\frac{y}{g}\frac{dr}{dt}-\frac{z}{g}\frac{dq}{dt}\right) \text{ etc.} \quad (12)$$

and the corresponding couples due to rotation take the form

$$A\frac{dp}{gdt}-F\frac{dq}{gdt}-E\frac{dr}{gdt}$$

$$-yM\left(\frac{dw^1}{gdt}+\frac{x}{g}\frac{dq}{dt}-\frac{y}{g}\frac{dp}{dt}\right)+zM\left(\frac{dv^1}{gdt}+\frac{z}{g}\frac{ds}{dt}-\frac{x}{g}\frac{dr}{dt}\right)$$

which reduce to

$$A^1\frac{dp}{gdt}-F^1\frac{dp}{gdt}-E^1\frac{dr}{gdt}-yM\frac{dw^1}{gdt}+zM\frac{dv^1}{gdt} \quad . \quad . \quad . \quad (13)$$

where

$$A^1=A+M(y^2+z^2) \qquad E^1=E+Mxz \qquad F^1=F+Mxy$$

so that A^1, B^1, C^1, D^1, E^1, F^1 are the moments and products of inertia about the new axes.

The equations, therefore, now assume the final forms

$$W(\dot{u}+y\dot{r}-z\dot{q})/g = S_x+\theta_3 S_y-\theta_2 S_z+W(l+\theta_3 m-\theta_2 n) - X_o-(u+\theta_3 V_2-\theta_2 V_3)X_u-\ldots-rX_r \quad . \quad . \quad (14)$$

$$\{A^1\dot{p}-F^1\dot{q}-E^1\dot{r}+M(z\dot{v}-y\dot{w})\}/g = W\{z(m+\theta_1 n-\theta_3 l)-y(n+\theta_2 l-\theta_1 m)\} - L_o-(u+\theta_3 V_2-\theta_2 V_3)L_u-\ldots-rL_r \quad . \quad . \quad (15)$$

V.—LATERAL OSCILLATIONS.

9. In this case the origin must be taken at O^1 and the equations can further be simplified by *taking the axis of u horizontal*, so that we have $V_2=O=V_3$, $l=o=n$, $m=1$. Also we may put $V_1=U$ where U is the actual velocity of the wind. Let β be the inclination of the string to the horizon. Putting $S_x=S\cos\beta$ and $S_y=S\sin\beta$, the equations give, with ζ, θ_1, θ_2, proportional to $e^{\lambda t}$

$$W\lambda^2(\zeta+x\theta_2-y\theta_1)/g = -W\theta_1-Z_w(\lambda\zeta+U\theta_1)-\lambda(\theta_1 Z_p+\theta_2 Z_q) - S\zeta/s+S_p+S(\theta_2\cos\beta-\theta_1\sin\beta) \quad . \quad . \quad . \quad (16)$$

$$\lambda^2(A\theta_1-F\theta_2-\zeta My)/g = Wy\theta_1-L_w(\lambda\zeta+U\theta_1)-\lambda(\theta_1 L_p+\theta_2 L_q) \quad (17)$$

$$\lambda^2(\beta\theta_2-F\theta_1+\zeta Mx)/g = -Wx\theta_1-M_w(\lambda\zeta+U\theta_1)-\lambda(\theta_1 M_p+\theta_2 M_q) \quad (18)$$

If we eliminate θ_1 from the second and third of these equations λ occurs as a factor of the resulting equation. Dividing out this factor we obtain an equation of the fifth degree for the period equation in λ.

10. CASE I.—*In the case of an infinitely long string* where $S\zeta/s$ is negligible we may put $\lambda\zeta=w$, and the displacement ζ does not occur in the equations by itself. In such a case the equilibrium is obviously neutral for displacements along the axis of z. The λ equation again reduces to the fifth degree, but as the eliminant of the second and third equations does not in general now contain λ as a factor, the equations for λ cannot be again reduced in degree.

For the purpose of studying the *oscillations of the kite itself* it is, however, better to take $s=\infty$, as if it should be necessary it would always be possible to ascertain the effects of shortening the string in the form of small corrections. The *stability of position* dependent on s being finite can be investigated separately by neglecting all but first powers of λ in the period equation.

11. CASE II.—*In the case of a plane kite without keels or auxiliary surfaces*, offering no tangential resistance, if we suppose α to be the inclination of its plane to the horizon, it is clear that $Z=O$ and $L\sin\alpha+M\cos\alpha=O$ for all displacements, the latter equation representing the fact that no couple can be set up tending to rotate the kite in its own plane.

In this case the equations simplify. For multiplying (2) by $\sin\alpha$, (3) by $\cos\alpha$ and adding and omitting the terms which vanish in (1) we have the two equations

$$W\lambda^2(x_2-y\theta_1)/g+W(\zeta\lambda^2/g+1)\theta_1 = S(\theta_2\cos\beta-\theta_1\sin\beta) \quad . \quad . \quad (19)$$

$$\lambda^2\{(A\theta_1-F\theta_2)\sin\alpha+(\beta\theta_2-F\theta_1)\cos\alpha\}/g + W(\zeta\lambda^2/g+1)(x\cos\alpha-y\sin\alpha)\theta_1 = O \quad . \quad . \quad . \quad . \quad . \quad (20)$$

These are evidently the equations obtained by resolving and taking moments in the plane of the kite. At the same time the equation apparently in λ does not usually reduce to a lower degree. If, however, there is no lateral displacement of the centre of pressure when the wind blows sideways on the kite L_w and M_w

will vanish, and the terms involving $\lambda\zeta$ will vanish. In this case λ^2 will occur as a factor of ζ throughout and dividing by this the equation reduces to a biquadratic.

CASE III.—*If the point of attachment is fixed,* $\zeta = o$ and 17, 18 lead to a biquadratic in λ.

VI.—LONGITUDINAL OSCILLATIONS.

12. In this case the origin, in the case of a forked string, must be taken at the fork, also it is important to take the axis of x along the direction of the string, and its inclination to the horizon being β we have

$l = \sin\beta$, $m = \cos\beta$, $n = o$.
$V_1 = U\cos\beta$, $V_2 = U\sin\beta$ where U is the horizontal wind velocity.

Also ζ, θ_1, θ_2, and their differential coefficients vanish, and the equations become on assuming ξ, η and θ_3 each proportional to e^{λ}

$$W\lambda^2(\xi + y\theta_3)/g = S_x + \theta_3 S_y + W(\sin\beta + \theta_3\cos\beta) - X_o - (\lambda\xi - \theta_3 U_o\sin\beta)X_u - (\lambda\eta - \theta_3 V\cos\beta)X_v - \lambda\theta_3 X_r \quad . \quad (21)$$

$$W\lambda^2(\eta - x\theta_3)/g = S_y - \theta_3 S_x + W(\cos\beta - \theta_3\sin\beta) - V_o - (\lambda\xi + \theta_3 U\sin\theta)V_u - (\lambda\eta - \theta_3 U\cos\theta)Y_o - \lambda\theta_3 Y_r \quad . \quad . \quad (22)$$

$$\lambda^2\left\{C\theta_3 + W(\xi y - \eta x)\right\}/g = W\left\{y(\sin\beta + \theta_3\cos\beta) - x(\cos\beta - \theta_3\sin\beta)\right\} - N_o - (\lambda\xi + \theta_3 U\sin\beta)N_u - (\lambda\eta - \theta_3 U\cos\beta)N_v - \lambda\theta_3 N_r \quad . \quad (23)$$

13. The conditions of equilibrium give with $S_x = S_o$ and $S_y = O$

$$S_o + W\sin\beta - X_o = O \quad . \quad . \quad . \quad (24)$$

$$W\cos\beta - \gamma_o = O \quad . \quad . \quad . \quad (25)$$

$$W(y\sin\beta - x\cos\beta) - N_o = O \quad . \quad . \quad . \quad (26)$$

in virtue of which the terms underlined vanish, the constant portion of the tension in S being cancelled in consequence. Taking the most general case of an extensible string of finite length we thus obtain

$$W\lambda^2(\xi + y\theta_3)/g = -(\lambda\xi + \theta_3 U\sin\beta)X_u - (\lambda\eta - U\theta_3\cos\beta)X_v - \lambda\theta_3 X_r - E\xi/s + W\theta_3\cos\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$W\lambda^2(\eta - x\theta_3)/g = -(\lambda\xi + \theta_3 U\sin\beta)Y_u - (\lambda\eta - \theta_3 U\cos\beta)Y_v - \lambda\theta_3 Y_r - S_o(\eta/s + \theta_3) - M\theta_3\sin\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$\lambda^2\left\{C\theta_3 + W(\xi y - \eta x)\right\}/g = -(\lambda\xi + \theta_3 U\sin\beta)N_u - (\lambda\eta - \theta_3 U\cos\beta)N_v - \lambda\theta_3 N_r + W\theta_3(y\cos\beta + x\sin\beta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

In the most general cases of an extensible string of finite length these lead to a period equation of the sixth degree, but the following modifications occur.

14. CASE I.—**String finite but inextensible.**—Here $\xi = O$, and we can write ΔS for $-E\xi/s$, where ΔS is the change in tension due to the small displacements and is small. In this case we must put $\xi = O$ in the second and third equations and these lead to a biquadratic equation for λ.

15. CASE II.—**String infinitely long and inextensible.**—Here $\xi = O$ as before, but η does not occur in the second equation without the factor λ. Replacing $\lambda\eta$ by v, λ only occurs in the first degree in the terms containing v and the resulting equation for λ is a cubic.

The extraneous solution $\lambda = O$ depends on the fact that equilibrium is neutral for changes of position of the kite in a direction perpendicular to the string.

If the string is very long the value of λ corresponding to the last mentioned solution is very small and hence the stability of position of the kite can be investigated approximately by neglecting all powers of λ above the first in (27), (29) and their eliminants. In this case we may regard the oscillations of the kite itself as given to a first approximation by the solution corresponding to $S = \infty$, the effects of the finite length taking the form of small corrections.

16. CASE III.—**String very extensible.**—The opposite case to the last one is that in which the string, besides being infinitely long, is very extensible, so that the term $-E\xi/s$ is very small. In this case stability of position is very slight for displacements parallel to either axes, and it can, as before, be investigated by only retaining the lowest powers of λ in the three equations, while the oscillations of the kite proper can now be determined approximately by neglecting both ξ/s and η/s and putting $\lambda\xi = u$, $\lambda\eta = v$. All three equations have to be used, but they involve linear functions of u and v and quadratic functions of θ, hence the eliminant is of the fourth degree in X. It is to be observed that in the second equation the term $S_0\theta_3$ is *not* to be neglected under any circumstances.

17. CASE IV.—**Point of attachment fixed.**—In this case $\xi = O$, $\eta = O$; equations (27) and (28) determine the tension components at any instant and (29) becomes a quadratic equation for the values of λ, the oscillations depending only on the co-ordinate θ_3.

There should now be no difficulty in discussing any further applications that may be of interest, such as the conditions of stability. There are, of course, other methods of forming the equations of small oscillation, and it will conduce to accuracy in such cases if the formulæ so obtained are compared and checked with the present ones. It will, however, be seen that Prof. Bose's formulæ do not agree with those of the present investigation, and that his period equations are not of the right degree.

Mr. W. E. H. Berwick has kindly checked the formulæ in this paper, and it is hoped that they are correct; failing this, that any unintentional slips may be discovered and corrected without difficulty.

THE VALUE OF "K."

[The course of the war, aeronautically, has so far shown that we have little to learn in aerodynamics from Germany. The following article is, however, published as being suggestive of openings for experiment (after the war) which might be undertaken at no great cost by private individuals in this country with possibly fruitful results. Much yet remains to be learnt in the field of aerodynamics, and it is clear that the increasing dedication of aeronautics to military uses is likely to clog progress in some directions, while greatly increasing it in others. It would be a great pity, therefore, if the private experimenter were to discontinue his activities because of the researches made on behalf of the national air services. He should, however, not be content with too crude methods, such for example as the timing apparatus used by Lilienthal, which is incapable of affording consistently accurate measurements of small time intervals. It cannot be said that the experiments are at all conclusive, but on the other hand, many objections can be raised to other determinations of "K," and it is very desirable that conclusive experiments should be carried out to elucidate the discrepancies that are to be found between the values assigned to "K" by different workers, especially where these discrepancies occur between a value from experiments in "still air" and a value from "moving air" experiments.—EDITOR.]

"RESEARCHES ON THE NORMAL AIR RESISTANCE = K."

BY GUSTAV LILIENTHAL.*

TRANSLATED BY W. R. TURNBULL, M.E., A.F.AË.S.

Some years ago, during my stay in Rio de Janeiro, I busied myself during spare time in determining the value of the normal pressure (by means of a plane mounted on a rotary arm) in such a way that no disturbing induced air current could arise.

For this purpose I had at my disposal a room 12 metres square by 8 metres high. I employed an apparatus by which a rotating arm 9 metres long described a quadrant or quarter circle. The test surface was 0.25 square metres in size. Since the surface traversed the quarter circle only once it encountered only still air. Uniform motion was controlled by the curve described by a vibrating spring on the end of which was fastened a small paint brush with colour. The spring was fastened one meter from the turning centre and vibrated over a paper spread on a measuring table. (Figures 1 and 2, not reproduced.)

The results were however not reliable enough on account of the shortness of the observations and the small size of the room. After my return to Germany, in the spring of 1913, I therefore undertook new experiments on a larger scale.

Through the kindness of the director of the South-West Electricity Works, there was placed at my disposal a large empty machinery hall in Schöneberg of 32 metres clear width, 60 metres long and 15 metres high.

In place of the earlier quarter circle run I built a semi-circular whirling arm whose radius, to the centre of pressure of the test surface, was 14.137 metres. This furnished a half-circle run of 45 metres. Fig. 3 shows the plan view and Fig. 4 the side elevation of the apparatus. Figs. 5 and 6 (not reproduced) are photographs of the same, the construction being as follows:—

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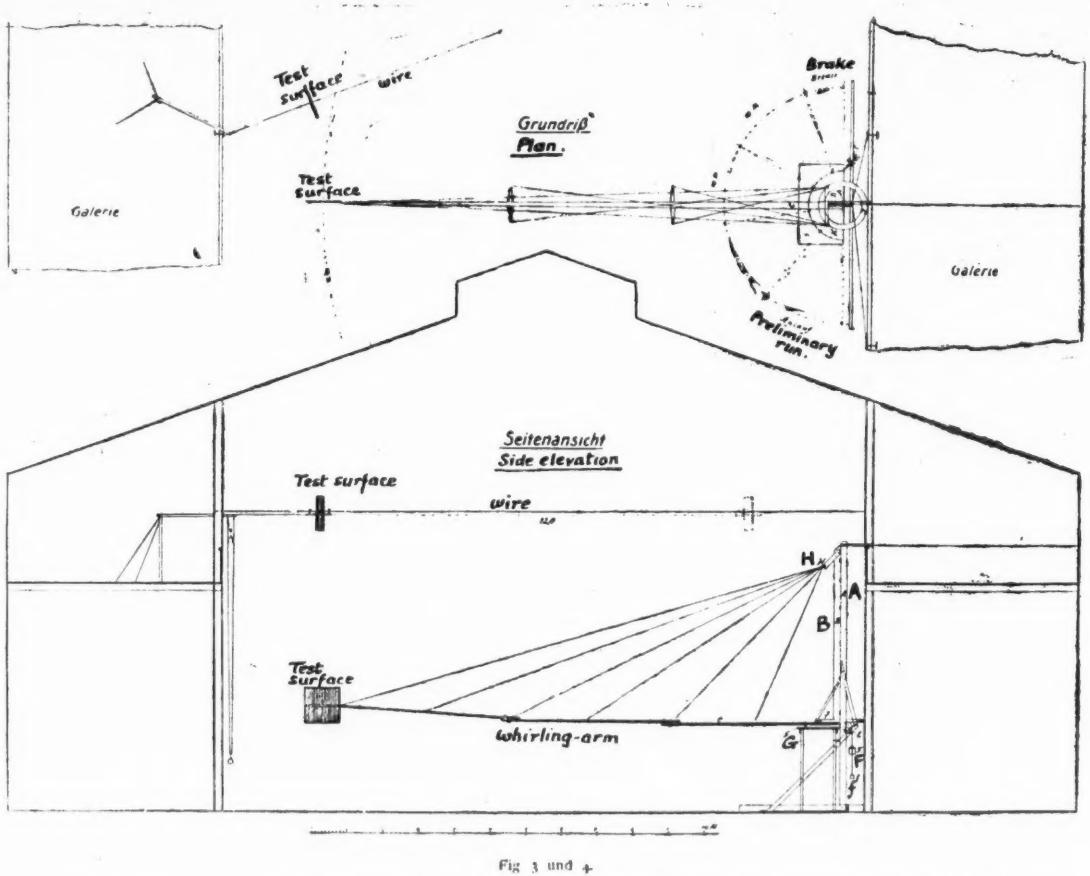


Fig 3 und 4

FIGS. 3 AND 4. Plan and elevation of arrangement of straight run apparatus and whirling-arm.

From a T-shaped post A, which is held vertical and in place by guys and supports from the edge of the gallery and side walls, the upright B is supported and turns on hinges. On the lower end of B the horizontal arm C is fastened and is supported by stay-wires from the crosshead H. The end of the arm C carries the test surface, whose centre of pressure is determined according to the centre of gravity of a truncated pyramid. The base of this pyramid is formed by the square of the outer trajectory of the test surface. The surface of the truncated side is equal to the square of the inner trajectory, the height of the truncated pyramid being the width of the test surface.

At the root of the arm, and supported on it, is a disc with grooved rim (with radius of 0.696 metre) whose centre lies in the axis of the rotating upright B. From this disc a wire cord leads over the pulley E and to a weight F. This pulley has a diameter of 35cm. and is ball-bearing, the frictional increase being negligible from no-load to full-load. Underneath F is hung a smaller weight f, by means of which the preliminary run, for overcoming the inertia, is made as short as possible.

It was found by trial that a preliminary run of 40° was necessary in order that the turning movement thereafter should be uniform. The speed of rotation was determined from a semi-circular arc, of one metre radius, graduated in degrees and attached to a measuring table G. The arm C turns above this arc and 10cm. from it. Little sticks of wood, with a base, are placed on the graduations of the arc and are knocked down by the arm in passing. In this way one can accurately measure the time that the arm takes to go from 40° to 160°, *i.e.*, 120°.

Since the semi-circular path of the test surface is 45 metres long, an angular movement of 4° is equivalent to one metre of path.

At first a test surface of 1 square metre was observed which was made to cover the angle of 120° (viz., 30 metres) in respectively 1, 2 and 3 seconds.*

For the adjustment of the uniform motion the times were taken for the passage of the arm from 40° to 80°, 80° to 120°, and 120° to 160°. (The German original being somewhat awkwardly worded for translation the following paraphrase is substituted.) The size of the weights was determined as follows:— A weight equivalent to that of the test surface was placed at the end of the arm and a trial run was made so that the motion between 40° and 160° was uniform. This gave the size of weights, F and f, necessary to overcome acceleration and to give when opposed to the air resistance of the arm itself a uniform speed over the trial arc. Then, when the test surface was in place, the weight that had to be added to F to give the same uniform speed over the 40° to 160° arc as attained in the trial run was equivalent to the air resistance of the test surface multiplied by the arm ratio. The so found weight has to be divided by the arm ratio, viz., by $\frac{14.137}{0.696} = 20.31$, and the so found quotient is then equal to the air resistance of the test surface. This value, divided by the square of the velocity per second and reduced to a standard temperature of 15° C., then gives the value of K. A further reduction to correct for the height of barometer was not applied since all measurements were made with barometer between 755 and 765 m.m.

The second test surface of 4 square metres could only be tested at a velocity of 1 metre per sec.

Table I. gives the results obtained. According to this the value for K averages 0.096 to 0.097 for the one square metre surface and 0.105 for the four square metre surface, with circular motion.

* Obviously this statement should read at the rate of 1, 2 and 3 metres per second; compare Table I.

TABLE I.

Semi-circular Path.

Date	Size of Surface. Sq. m.	Nett loading Kg.	Observed time for 30-metre run. Secs.	Average Time. Secs.	Velocity in metres. Per sec.	Temperature.	"K" at 15° C.	Average value of "K."
25.6.13	1.0	1.960	31.0					
			30.8					
			29.6					
			30.2	30.01	1.0	16°	0.097	
			30.0					
			29.6					
			30.0					0.096
13.10.13	1.0	1.950	30.0					
			30.0					
			30.0	30.0	1.0	11°	0.095	
			30.0					
			30.0					
13.10.13	1.0	7.950	15.0					
			15.0					
			15.0					
			15.0					
			15.0					
			15.0					
			15.0					
			14.8	14.99	2.0	10°	0.096	0.096
			14.9					
			15.1					
			15.0					
			15.0					
			15.0					
			15.0					
18.10.13	1.0	18.750	10.0	10.0	3.0	8°	0.100	
19.10.13	1.0	16.850	10.5	10.55	2.84	6½°	0.099	
			10.6					
19.10.13	1.0	17.350	10.1					
			10.3	10.32	2.90	6½°	0.098	
			10.6					
			10.3					
19.10.13	1.0	17.650	10.0					
			10.4					
			10.0					
			10.4					
			10.4					0.097
			10.1	10.21	2.93	8°	0.098	
			10.2					
			10.2					
			10.2					
			10.2					
19.10.13	1.0	18.150	9.7					
			10.0					
			10.0					
			10.0	9.97	3.01	8°	0.096	
			10.1					
			10.0					
			10.0					

Date.	Size of surface. Sq. m.	Nett loading Kg.	Observed time for 30 m. run. Secs.	Average time. Secs.	Velocity in metres. Per sec.	Temperature.	"K" at 15° C.	Average value of "K."
19.10.13	1.0	18.300	10.1					0.097
			9.9					
			10.0					
			10.0	10.0	3.0	9°	0.098	
			9.9					
			10.1					
			10.0					
19.10.13	1.0	18.450	9.8					
			10.0					
			9.9	9.92	3.02	9°	0.097	
			10.0					
10.11.13	1.0	18.296	10.0					
			10.0	10.0	3.0	8°	0.098	
			10.0					
15.11.13	1.0	18.220	10.1					
			10.1	10.1	2.97	7°	0.098	
			10.1					
15.11.13	1.0	18.520	10.0					
			10.0					
			10.0					
			10.0					
			10.0					
			9.9	10.0	3.0	7½°	0.098	
			10.1					
			10.0					
			9.9					
			10.1					
			10.0					
15.11.13	1.0	17.920	10.3					
			10.1					
			10.3					
			10.0	10.2	2.94	8°	0.098	
			10.3					
			10.3					
			10.1					
15.11.13	1.0	16.420	10.6					
			10.4	10.5	2.85	8½°	0.096	
			10.5					
15.11.13	1.0	13.885	11.3					
			11.3	11.3	2.65	8½°	0.095	
15.11.13	1.0	18.440	10.0	10.0	3.0	7½°	0.098	
3.8.13	4.0	8.570	30.0	30.0	1.0	21°	0.107	
23.8.13	4.0	8.500	30.4					
			30.6					
			30.2					
			30.2	30.3	0.99	20°	0.104	
			30.1					
			30.2					0.105
8.9.13	4.0	8.565	30.0					
			30.2					
			30.0	30.0	1.0	17°	0.106	
			29.8					

I have besides made straight-line towing experiments, on a surface of 1 × 1 metres, the results of which are shown in Table II.

TABLE II.

Straight-line Path.

Date.	Size of surface. Sq. m.	Nett loading Kg.	Observed time for 12 m. run. Secs.	Average time. Secs.	Velocity in metres. Per sec.	Temperature.	"K" at 15° C.	Average value of "K."
13.7.13	1.00	0.402	6.0	6.0	2.0	25.5°	0.103	0.102
24.7.13	1.00	0.400	6.0	6.0	2.0	20°	0.101	
10.11.13	1.0	0.950	4.0	4.0	3.0	8°	0.103	
10.11.13	1.0	0.925	4.0	4.0	3.0	8°	0.102	
15.11.13	1.0	0.875	4.2 4.3 4.2 4.2 4.2 4.2 4.2 4.2 4.2 4.2	4.21	2.90	8°	0.102	
15.11.15	1.0	0.950	4.0 4.0 4.0 4.1 4.0 4.1 4.0 4.0 3.9 4.0 4.0	4.01	2.99	8°	0.104	0.101
15.11.13	1.0	0.984	3.8 3.8 3.8 3.8	3.8	3.16	8°	0.096	
15.11.13	1.0	0.712	4.7 4.6 4.6 4.6 4.5 4.6 4.7 4.6 4.6 4.5 4.5 4.6	4.59	2.61	8°	0.103	
5.1.14	1.0	0.921	4.0 4.0 4.0	4.0	3.0	$3\frac{1}{2}$°	0.098	

The arrangement is shown in Figures 3 and 4 and photographs of the apparatus are shown in Figures 7 and 8 (not reproduced). A strong wire, 5m.m. in diameter, is very tightly stretched between two columns of the gallery, obliquely through the middle nave of the hall, so that there is a clear run of 20 metres. Of this distance 4 metres was required for the preliminary run and 4 metres for braking, leaving a distance of 12 metres for observation. On a rod, supported by two pulleys running on the wire, the test surface was fastened.

During the calibration run, in place of the test surface, a corresponding weight was attached to the rod, which was carried along the wire by means of a small cord, running over pulleys, to which weights were attached. The time of the run was determined by a stop-watch.

The values of K for 2 and 3 metres per sec. amounted to 0.102 and 0.101kg. respectively. For a correction of increased friction of the pulleys, with full load, 1% is allowed.

On the basis of these experiments I adopt for my value of K, 0.1kg.

I am aware that Eiffel, by means of falling experiments, obtained a value of only 0.066kg. for K. However, in these experiments, which were undertaken in the open air from the scaffold of the Eiffel Tower, several automatic recording devices were used so that one can conclude that the pressure on the comparatively small surfaces was considerably reduced.

Dr. Föppel gives K = 0.075kg. for the result of his experiments undertaken at Göttingen. He points out, in his doctor's thesis, that this result approaches closely to the result obtained by Langley, viz., K = 0.080, using the rotary arm method.

On the question of the disturbing influence of the induced air current, when using the rotary arm method, I have carried out special experiments. I built a small rotary arm apparatus, turning in the vertical, that was very sensitive (Figs. 9, 10 and 11). In Table III. the results of the two series of experiments are given, the surface used being 8 × 20c.m., the area, after a deduction for the framework, being 138.2 sq. c.m.

TABLE III.

Measurement of the Induced Air-current with vertical rotary path.

Diameter of path of centre of pressure = 1.05.
Nett surface = 0.01382 sq. m.
7 revolutions with constant driving weight.

TABLE I.

First series. 11 experiments.

	1 revolution. Secs.	2 revolutions. Secs.	3 revolutions. Secs.	4 revolutions. Secs.	5 revolutions. Secs.	6 revolutions. Secs.	7 revolutions. Secs.
1.	11.0	19.4	29.8	37.8	47.2	56.8	66.8
2.	10.4	20.2	29.8	38.4	49.4	56.8	66.2
3.	10.4	19.2	29.8	38.4	47.4	57.8	65.4
4.	10.8	19.6	29.6	38.6	47.8	56.8	65.8
5.	10.6	19.6	29.2	38.4	48.4	57.4	66.8
6.	11.0	20.2	29.6	38.8	48.8	56.4	66.8
7.	10.8	20.4	29.6	38.8	48.4	56.8	65.6
8.	11.6	20.0	29.2	38.4	48.2	56.4	66.8
9.	10.6	20.4	29.2	37.8	48.4	57.6	66.4
10.	10.6	20.2	28.6	37.8	47.8	57.2	66.4
11.	11.0	20.4	29.2	38.8	47.4	58.4	66.6
Sum	118.8	219.6	323.6	422.0	529.2	628.4	729.0 = 11 revs.
11 :=	10.8	19.96	29.4	38.4	48.1	57.4	66.2 = 1 rev.
Difference 10.8		9.98	9.80	9.65	9.62	9.56	9.45 Sec.

TABLE II.

Second series. 10 experiments.

	1 revolution. Secs.	2 revolutions. Secs.	3 revolutions. Secs.	4 revolutions. Secs.	5 revolutions. Secs.	6 revolutions. Secs.	7 revolutions. Secs.
1.	9.2	16.4	24.2	31.0	38.4	46.2	50.8
2.	8.6	15.6	24.2	31.6	39.0	46.0	52.6
3.	8.6	16.6	23.8	31.8	38.8	45.4	51.4
4.	8.6	16.0	24.0	31.8	39.4	46.6	52.6
5.	8.8	16.6	23.6	32.2	39.2	45.8	53.8
6.	8.8	16.8	24.2	32.8	39.4	45.8	54.2
7.	8.6	16.6	23.8	31.4	40.0	47.4	52.6
8.	8.0	16.2	24.0	32.2	38.6	46.4	51.6
9.	9.2	16.8	23.6	30.8	40.2	47.4	53.4
10.	9.0	16.4	24.2	31.8	39.2	46.4	52.6
Sum	87.4	164.0	239.6	317.4	392.2	463.6	525.6 = 10 revs.
	8.74	16.40	23.96	31.74	39.22	46.36	52.56 = 1 rev.
Difference	8.74	8.20	7.99	7.93	7.84	7.72	7.51 Sec.

In these experiments the driving weight was kept constant and the time was recorded for the first and the following six revolutions, in such a way that the increase of velocity, from one revolution to the next, could be obtained by subtraction.

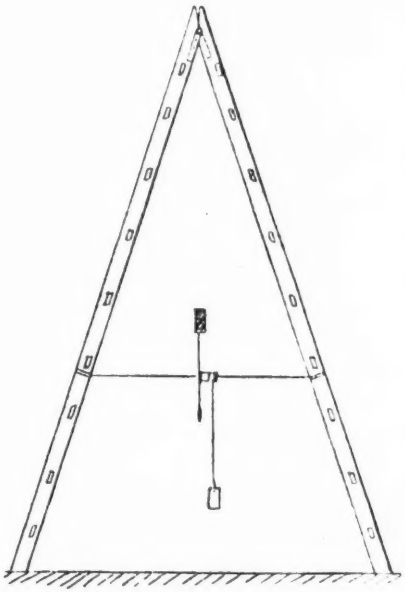
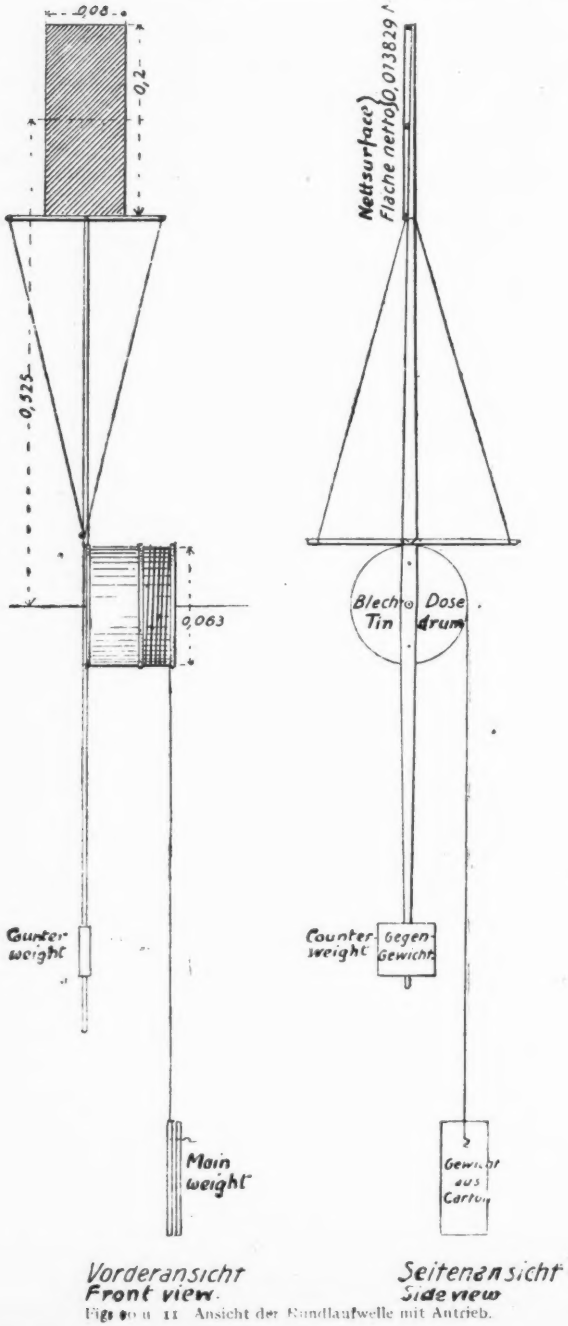


Fig. 9. Gesamtanordnung

FIG. 9. General arrangement of vertical whirling-arm.

Before operating with the test surface the apparatus was calibrated without the surface, the weight of the surface being at first fastened, by a little piece of sheet metal, at the centre of gravity of the surface frame. Since the apparatus



Figs 10 u. 11. Ansicht der Rundlaufwelle mit Antrieb.

Kleiner Rundlauf.

FIGS. 10 AND 11.
Vertical whirling-arm, details.

was extraordinarily sensitive it was necessary to apply to the frame the few drops of " syndetikon " with which the paper surface was glued. Otherwise, this small weight would have introduced an error. With this preparation, the driving weight, to attain the desired velocity of rotation for the " empty run," was determined. With the surface attached and plotting the time of single revolutions at equal intervals as ordinates and corresponding lengths of path as abscissæ and drawing a smooth curve through the tops of the ordinates we obtain a ⁀ shaped curve. (See Figure 12.)

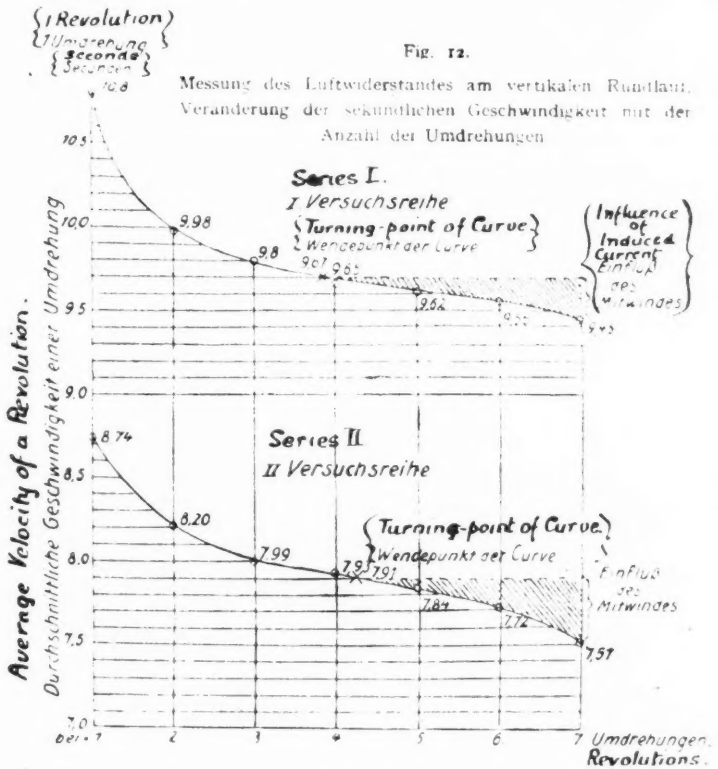


Fig. 12.

Messung des Luftwiderstandes am vertikalen Rundlauf. Veränderung der sekundlichen Geschwindigkeit mit der Anzahl der Umdrehungen

FIG. 12.

Measurement of air resistance with vertical rotary path.
Alteration of the velocity per second with the number of revolutions.

This curve is at first quite steep corresponding to the overcoming of the inertia. The acceleration attains a minimum, at the crossed point, and then increases at first slowly and then more rapidly.

The turning point of the curve lies very nearly at the same point of time at which the apparatus, on the " empty run," arrives at uniform motion.

The increased inclination of the curve shows clearly the influence of the induced air current even with the smaller velocities in which the test surface takes 7 to 8 seconds to return to the same point.

If one uses high velocities the circular run requires a longer time for the

preliminary run before uniform motion is attained. In fact the apparatus, running empty, shows the more induced current the oftener the rotations occur. The motion can only become uniform when the induced current attains a maximum. This occurs according to the size and form of the space in which the experiments are undertaken.

Prof. Langley's experiments were undertaken under the auspices of the Smithsonian Institute, at the Alleghanny Observatory, which is located at a high altitude. Langley used a whirling arm apparatus, driven by a steam engine, with which velocities up to 45 metres per sec. were attained.

The measuring apparatus was produced, at great cost, in the course of four years. The circular path of 9 metres radius was installed in the open air, in order that the induced air current, which arose during the operation of the apparatus, would be as small as possible. It was soon found that a perfectly calm day rarely occurred, so a fence, 2 metres high, was built about the experimental grounds.

The test surface consisted of a square of one foot on the side (= 0.0929 sq. metres). The middle of the surface was taken as the centre of pressure, while in reality the centre of pressure would lie 3m.m. out from the middle.

Measurements took place on three days; on the first day there was a wind of 0.52 metres per sec., on the second day a wind of 0.37 metres per sec., and on the third day a stronger and gustier wind. The measurements of the third day must therefore be thrown out. On the first day 6, and on the second 8, measurements were taken, in which the test surface was at right angles to the line of motion. In the other measurements the surface was placed at angles of 7.5, 15, 30 and 45 degrees.

The results given in two tables, which are all reduced to metres and kilograms, show the normal pressure, = K, for one square centimetre in grams and for one metre per second velocity. K refigured for one square metre in kilograms and for the following velocities is here given:—

First day at	4.47	metres	per sec. velocity	K = 0.097 k.g.
,,	4.47	,,	,,	K = 0.092 ,,
,,	8.66	,,	,,	K = 0.066 ,,
,,	8.57	,,	,,	K = 0.067 ,,
,,	9.85	,,	,,	K = 0.075 ,,
,,	10.50	,,	,,	K = 0.070 ,,

at 21° Celsius and 736 m.m. barometer.

Langley adopted the average value for K, namely, 0.078 kg. After reduction to 10° Celsius and a barometer of 760 m.m. (the position of the observatory is 400m. above sea-level) the corresponding value of K works out at 0.081 kg.

On the second day the measurements were taken at 19.4° C. as follows:—

4.48	metres	per sec. velocity	K = 0.090 k.g.
4.48	,,	,,	K = 0.088 ,,
8.77	,,	,,	K = 0.075 ,,
8.67	,,	,,	K = 0.075 ,,
11.20	,,	,,	K = 0.074 ,,
10.59	,,	,,	K = 0.070 ,,
9.77	,,	,,	K = 0.072 ,,
9.78	,,	,,	K = 0.070 ,,

These give an average value for K of 0.077 or after reduction to normal temperature and barometer level K = 0.079 kg.

These two values of 0.081 and 0.079 gave Langley a mean value of K = 0.080 k.g. which he adopted. This value of K is very nearly the same as

German experimental stations have established, as Dr. Föppel points out in his doctor's thesis.

By closer observation of Langley's measurements, I cannot help doubting the reliability of the determined value for K. From both tables it is clear *that with the smaller velocities K is considerably larger than with the greater velocities.* This can only be a result of the induced air current, which is inseparable from the whirling arm method. The preliminary run, which is necessary for the attainment of uniform motion, must undoubtedly set up a motion of the surrounding air, even when the experiments are conducted in the open air. On the first day the determination of K fell from 0.097 to 0.066 and on the second day from 0.090 to 0.070.

With a velocity of 10 metres per second the test surface made a complete revolution in 5.6 seconds. Even when the test surface is relatively small a very considerable turbulence of the whole surrounding air is set up, on account of the air resistance of the whirling arm and its supporting guy wires. Even with the low velocity of 4.47 metres per second, a complete revolution being made in about 12 seconds, the induced air current, although much smaller, is still present.

According to my way of thinking Langley's results for the lowest velocities are the best to accept and the value of K should even then be somewhat increased.

Langley points out, in the introduction to his treatise, that it would have been better to carry out the investigations in an enclosed room, but he feared the increased influence of the induced air current, since, in an enclosed room, the surrounding air is set in stronger motion than in the open air.

The further experiments of Langley, with inclined surfaces, and the determination of the lift and drift, are in a similar way unfavourably influenced as in the normal pressure measurements, also in these he neglected the shifting of the centre of pressure, beyond the centre of gravity, due to the circular path.

The " Hütte " gives as a value for K only 0.062 k.g. This small value was only theoretically obtained. One considers what occurs, creating a normal pressure, when a surface moves under the assumption that a surface of one square metre, in a second of time, sets a cubic metre of air in motion, and we then say that the air resistance corresponds to the formula $\frac{v^2\gamma}{2g} = \frac{1^2 \times 1.226}{2.981} = 0.0625$.

This consideration seems so clear and simple, but it does not in the least represent the actual process which takes place when a surface moves at right angles to an air current. I have tried to make the actual proceeding evident by special experiments.

Through the middle of my test surface of 1×1 metres, mounted on the arm, which moves in a semi-circle, I passed a rod, at right angles to the surface, and extending 2 metres in front and 2 metres behind the surface. Upon this rod (Figure 13, AA) at a distance of 10 c.m. before and behind the surface another rod BB was laid gradually across and held in position. Upon the rods BB were then brought a row of small vanes, spaced 10 c.m. apart, these vanes standing at right angles and capable of swinging. They carried round with them a small wire clamp which remained in position after the vane turned back. In this way the position of the vanes was determined which they had attained for that case in which the running out of the induced current left them. For each experiment only one row of vanes was placed before and behind the surface and then the rod was placed another 10 c.m. away, until it finally reached 2 metres from the surface. The positions which the vanes assumed being carefully noted. After the experiment all the vane settings were shown on a drawing (Fig. 13) and thus the stream lines were made visible.

Afterwards the experiments were repeated, for the vertical position, in which the rods BB were placed vertical to the rod AA. The vanes in this case could swing about a horizontal axis. The so found streamlines are shown in Figure 14.

One sees, from Figs. 13 and 14, that by no means only one cubic metre of air is set in motion, but considerably more. The experiments were carried out with a velocity of 4 metres per second. Both before and behind the surface

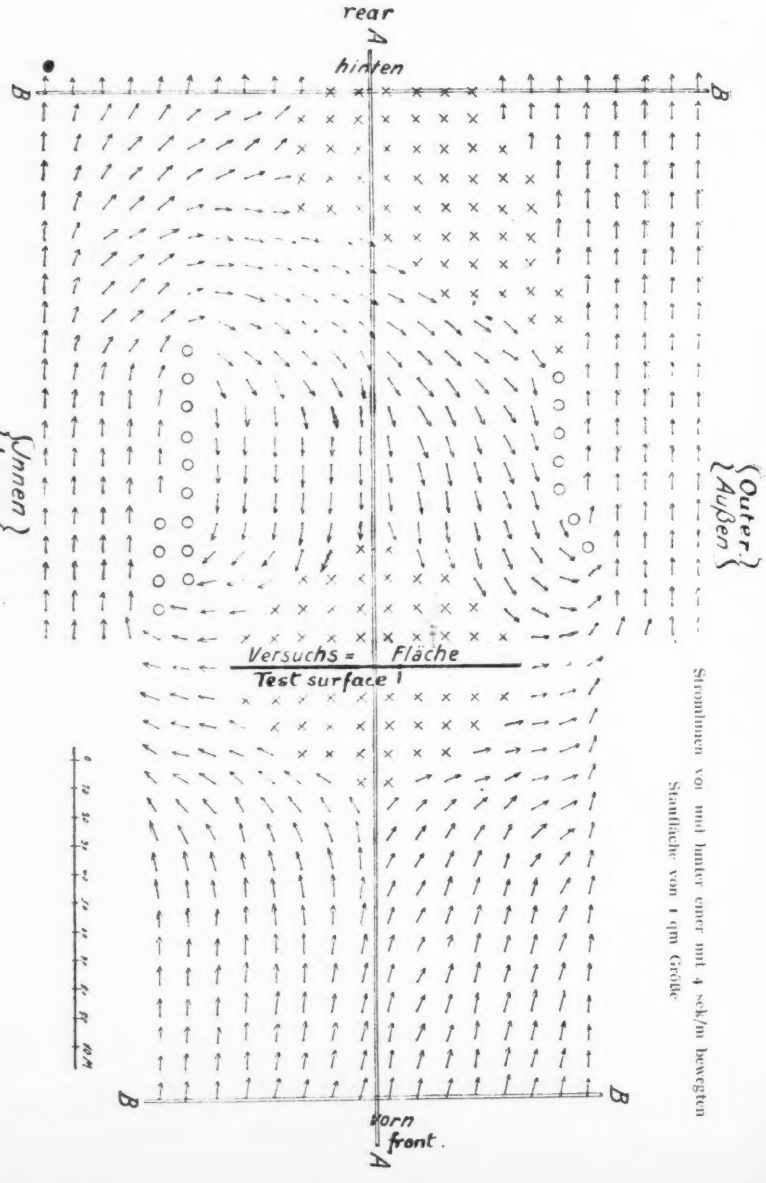


Fig. 13. Streamlines before and behind a surface of 1 sq. metre, moving at 4 metres per second.

Stromlinien vor und hinter einer mit 4 sek/m bewegten Standfläche von 1 qm Größe

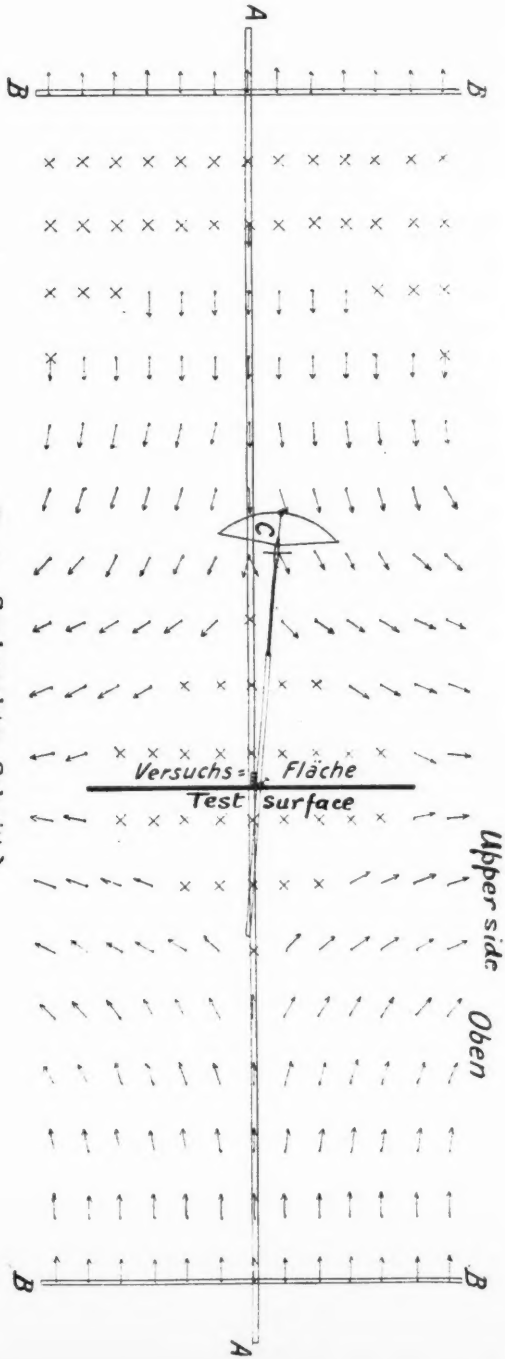


Fig. 14 Senkrechter Schnitt. Vertical Section.

there is formed a neutral zone, in the form of a pyramid, whose height equals the half-width of the surface. In this zone the vanes were not disturbed, but retained the position given to them at the beginning of the experiment.

At a distance of 1.50 metres behind the surface, the same phenomenon is seen, but the neutral zone has a different shape. It also makes a form, that enlarges the pyramid behind the surface to a right angled body. Then behind this zone the vanes again turn towards the rear.

The cause of this phenomenon is found in the fact that behind the surface there a region of decreased pressure towards which the air, further backwards, is drawn. Accurate demonstration, on the relations of pressure, could only be accomplished by manometric measurements. But in order to carry out such experiments it would be necessary to have the test surface stationary and use an air current against it.

I measured the strength of the incoming air stream by means of a pressure disc (C, Fig. 14) with spring scale, which was mounted behind the test surface that was carried on the arm in the semi-circular apparatus. The pressure disc, of 0.01 sq. metres size, attained a pressure of 7.5 grams (with a velocity of test surface of 4 metres per second), which corresponds to a velocity of 2.73 metres per second, adopting my value of K.

The inrushing air stream behind the test surface had therefore an absolute velocity of $4+2.73=6.73$ metres per second. One can thus see that such a complicated process cannot be explained by the simple displacement of a cubic metre of air, more especially as other matters, including the friction between contiguous masses of air, would have to be taken into consideration.

According to the streamlines, shown by the vane settings, the motion of the air is not a circular eddy motion, as the " Hütte " represents, but the streamlines behind the surface are, for the most part, straight line and turn, before the surface, sharply about. Only at a distance of about 10 c.m. do the vanes attain a rotary motion.

The pressure disc used for the pressure measurements is shown in Figure 15.

A short celluloid tube plays freely over a glass tube, which carries a scale on the inside. To this celluloid tube the pressure disc (10×10 c.m.) is so fastened that it is perpendicular when the glass tube is given a slight inclination to the horizontal. This inclination being so chosen that the friction of the celluloid tube is overcome.

A fine steel wire, of 0.5 m.m. diameter, is fastened forward of the glass tube, and bent into a bow by means of a thread attached to the ends. This thread is then attached to the celluloid tube. The scale, in the glass tube, is then calibrated by loading the surface with known weights, the weight of the surface and celluloid tube being allowed for.

The position that the pressure disc takes up, before the apparatus is braked, is shown by a small iron ring that is pushed along the glass tube by the celluloid tube.

This instrument behaved well so that I used it likewise for the measurement of the wind strength.

By a comparison of the streamlines, in Figures 13 and 14, one finds that symmetrical movements, upwards and downwards, take place; while in Figure 13 a stronger outward flow is shown towards the periphery of the circular path. This partly sidewise flow of the air must introduce a lessening of the final result. Consequently, the value of K obtained with circular motion must be less than that obtained with straight line motion.

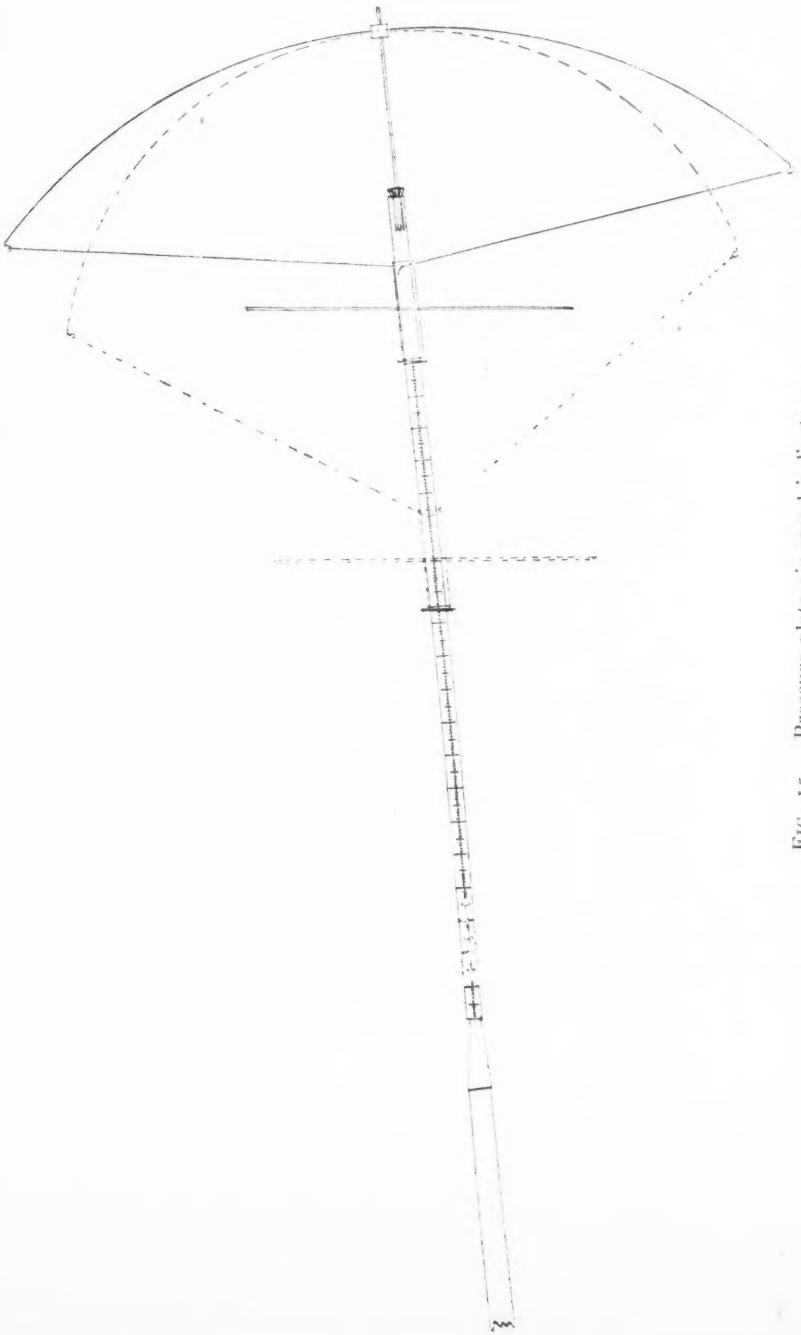


FIG. 15. Pressure plate air-speed indicator.

This difference amounted in my experiments to 3 or 4%, although the arm of my semi-circular apparatus had a radius of more than 14 metres. With smaller radii, the differences would be greater.

In order to work with larger surfaces it would be necessary to raise the test surface higher above the floor, so that the results would not be disturbed, and this would increase the difficulties of measurement.

Translator's Note.—It is doubtless very desirable to have it settled, once for all, whether the value for K is the same when a surface moves through still air as when a column of moving air impinges on a stationary surface.

Any method by which rotary motion is adopted, even for a semi-circle, is open to doubt, for questions of the centrifugal action of the air in contact (and before contact) with the test surface vitiates the result, even though we eliminate the influence of the induced air current.

Personally, I am a great believer in a straight line method, and Lilienthal, in the above article, has used this method, as well as a semi-circular method, but I fear his time measurements are open to question as he used a stop-watch and the time intervals measured were of the order of 4 to 6 seconds. As is well known, a stop-watch is open to objection, as there is usually an instrumental error *and* a personal error which *may* be additive; a much better system would be the employment of electrical contacts registering on a chronograph for the time measurement, with a very careful determination of the influence of friction of the rollers on the stretched wire and the pulley sheaves, etc.

The foregoing article is *suggestive* as to method, but leaves much to be desired as an accurate determination of K. In the semi-circular apparatus the arm has a large radius, which is of course desirable, but it is likely that in this case also (although not definitely stated) a stop-watch was employed.

Since the accurate determination of the time, and hence the velocity, is essential (more particularly since the velocity enters the equation as a *square*) a doubt is naturally thrown on the value of K.

As an interesting comparison using a straight line method, we look to the determination of K by the Duc de Guiche ("Essai d'Aerodynamique du Plan," 1911) and find that his value of K worked out at 0.074, which is in far closer accord with the results of other experimenters (using wind channel methods, etc.) than Lilienthal's figure of 0.101. Another straight line method, used by Eiffel with discs falling from the Eiffel Tower, gave K = 0.066; it is therefore probable that Lilienthal's figure for K, owing possibly to inaccuracies in time measurement or some other cause, is too high, but his methods are suggestive and it is desirable that his straight line method be repeated with accurate time measurement and frictional losses carefully determined and allowed for.

W. R. TURNBULL

"THE AERONAUTICAL INSTITUTE OF GREAT BRITAIN."

Professor G. H. Bryan, F.R.S., has asked that we should make it known that he has resigned from the "Executive Committee" of the above-named body, an announcement of the formation of which recently appeared in the Press. He states :—"When the Secretary invited me to serve on the Committee and enclosed "a prepaid telegraph form for reply, I certainly did not realise the character of the "Committee, and I do not think he could have realised the nature of my quali- "fications and disqualifications. Had the proposed Committee been a large and "thoroughly representative body my name might not have been out of place on "the list, but I cannot think that an executive committee containing *two* mathema- "ticians out of a total of five is likely to appeal to those responsible for the "manufacture and production of aircraft."

We are debarred from expressing any opinion on the newly-formed "Institute," but in the interest of aeronautics it is desirable that the following opinions expressed by the technical Press should be placed on record here, more especially as this Society was asked to nominate representatives on the Committee of the Institute, and thus identify itself with the aims of its promoters. (The Society did not accede to the request) :—

"The Aeroplane," *October 6th*, 1915.— . . . . . "As regards the "Maude-Desbleds effort, it seems at best unnecessary. At worst it is an "impertinence. For, apart from the fact that no one whose name has "appeared in connection with it has any obvious qualifications to teach either "the Government, the industry or the workmen anything connected with the "use or construction of aircraft, there already exist bodies which could do "the work if it were needed.

"The Aeronautical Society is the recognised technical authority, and "though it has been in a state of suspended animation since the outbreak of "war, it can be resuscitated whenever it is needed. . . . . On the "whole, I think the aircraft industry will do well to fight shy of the "'Aeronautical Institute of Great Britain' till it can prove that it is . . . . "a serious scientific institution approved by those controlling the Flying "services, and regarded with favour by the Councils of the Royal Aero Club "and of the Aeronautical Society."

"*October 20th*, 1915.— . . . . . "it appears . . . . . that "the Institute's latest idea is to poach on the preserves of the Royal Aero "Club and of the Aeronautical Society while the active members of those "established and trustworthy institutions are busy in the country's service. "One may therefore repeat the warning already given in this paper that those "who have the welfare of aviation at heart would do well to fight shy of the "'Institute.'"

"Aeronautics," *October 13th*, 1915.—"On the last day of September, as "was announced last week, the Royal Society of Arts entertained under its "hospitable roof a truly imposing conglomeration of talent, which proceeded "to arrogate to itself the grandiloquent title of 'The Aeronautical Institute.'

"To those of us who have watched the growth of aeronautics from its "puberty, if not from its infancy, the list of those who attended, or at any "rate 'had accepted the invitation to attend,' reads like a grotesque joke. "We see represented all classes of interests, from the Temple to the House "of Commons, including the Royal College of Surgeons, but we look in vain "for the name of any individual even remotely connected with present-day

"aeronautics, practical or scientific . . . . . Some of the objects of "the Institute are quite definitely and specifically provided for by the declared "activities of the Aeronautical Society, the Royal Aero Club or the Aerial "League. . . . . Apart from ethical considerations, it seems "presumptuous in the extreme for a mushroom organisation of aeronautical "nobodies to endeavour to usurp the work of the Society at the moment "when its hands are tied by the absence of most of its prominent members "on active duties in connection with the production or employment of "aircraft." . . . . .

"Flight," 10*th December*, 1915.—"By way of the opposite of all that "can be commended, is the extraordinary persistence of those interested in "the concern which has taken unto itself the very ambitious title of 'The "Aeronautical Institute of Great Britain.' When this so-called 'institute' "was brought into the world some few weeks ago, we recorded the inaugural "date of its 'birth' in such manner as to convey to all those who have any "judgment of their own our opinion of its merits. We hardly conceived that "anybody could take the scheme seriously, having regard to the already "existing and long-tried bodies in power in the world of Aviation. And so "we left the whole business to come by its end in a nice, quiet, natural way. "It would appear, however, as if there were still an inclination to proceed "with the attempt to force this 'institute' upon an unwilling and unsym-"pathetic world, judging by certain communications which reached this "office last week." . . . . .

NEW BOOKS AND PUBLICATIONS.

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Tenth Annual Report of the Meteorological Committee for the year ended 31/3/15. London : H.M. Stationery Office. 1915. pp. 93. Plates. Price, 5½d.

The Petrol Engine. F. J. Kean. London : 1915, E. & F. N. Spon. pp. 124. Figs. Price, 4s.

The Principle of Least Action. Philip E. B. Jourdain. Chicago & London : 1913. The Open Court Publishing Co. pp. 83. (Reprint from "The Monist," April and July, 1912, and April, 1913).

On the Equations of Motion of a Plane Surface through Air, with Special Reference to its Stability (Constrained Motion of a Kite). Professor J. M. Bose. Calcutta : 1914. The Presidency College, Calcutta. pp. 56. Figs.